Correction to Coherent Scattering Model for a Self-Affine Surface

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The coherent scattered electric field from a rough fractal surface is derived by exploiting the fact that concentric annuli about any chosen point have an rms height, ξ, that is related to the radius of the annulus, r, the rms slope at the wavelength scale, s_λ, and the Hurst exponent, H:

$$\xi(r) = \frac{\lambda}{\sqrt{2}} s_\lambda \left(\frac{r}{\lambda}\right)^H \tag{1}$$

The coherent scattered field for each annulus is:

$$E_{annulus}(r) = \exp\left[-2k^2 \xi(r)^2 \cos^2\theta\right] E_o \tag{2}$$

where $k=2\pi/\lambda$ and E_o is the incident field. The total scattered field is the integral over all annuli <u>within a given illuminated area</u> (which will not, in general, extend to infinity).

$$E_s = E_o\left[ie^{-ikZ}\right]\frac{2\pi\lambda R_e}{Z}\int_o^{\hat{r}_{\max}} \exp\left[-4\pi^2 s_\lambda^2 \hat{r}^{2H}\cos^2\theta\right]\hat{r}J_o(4\pi\hat{r}\sin\theta)\,d\hat{r} \tag{3}$$

where we introduce the dimensionless variable $\hat{r} = r/\lambda$. $\hat{r}_{\max}$ is thus the radius of the illuminated area in multiples of the radar wavelength, λ. J_0 is a Bessel function of the first order, θ is the radar incidence angle, and R_e is an effective reflectivity. If we let the integral in (3) be given by K, the coherent backscatter coefficient for the fractal surface is:

$$\sigma^o_{coherent} = \frac{16\pi^2 R_e^2 K^2}{\hat{r}^2_{\max}} \tag{4}$$

This represents a correction to Equations 5.30-5.31, which do not properly normalize for the illuminated area. In general, coherent echoes are unlikely to contribute to near-nadir returns unless the radar wavelength is on the order of many meters (e.g., radar sounder systems).